# Assignment Week 6. Options arbitrage

Data: You will receive a .csv file with market data for a stock and six options listed on that stock. The options consist of three puts and three calls, all sharing the same expiry but with different strike prices. The time to expiry in years, equal for each of the options, is given as a separate column called “TimeToExpiry”. The names of the other columns are given according to the format “BidPrice-P70” for the bid price of the put with strike 70, “AskPrice-C80” for the ask price of the call with strike 80, or “AskVolume-Stock” for the ask volume of the stock, etc.

Code: For this assignment, you may make use of the pre-supplied implementation of the Black-Scholes option value and delta calculations and the skeleton notebook file with starting points.

Assignment: Design and backtest an options delta-arbitrage trading strategy. Your strategy should seek to enter trades that, after being delta hedged with the stock, still result in a profit.

Throughout the assignment, you may assume a 0.0% interest rate () and a constant implied volatility () of 20%.

As the volatility and interest rate are known and constant, and the strike and time to expiry follow from the dataset, the only input to the Black-Scholes option pricing formula that is not uniquely determined is the underlying stock price .

A trade in an option carries delta-risk: the price of the option, and thus the profitability of the trade, varies with the underlying stock. By delta-hedging the option that risk is diminished.

Determine at each time step, what the price would be below (above) which you could buy (sell) the option, hedge the trade in the underlying stock directly, and still make money. Which stock price you should hedge against will depend on the sign of the delta of the option, as well as the side of the trade (buy/sell). The price you would be willing to buy at should be lower than the price you would be willing to sell at. How do these values compare to the market bids and asks? Are there opportunities to trade? Take them.

Ensure your total delta position stays below 20 at all times.

If you trade multiple options simultaneously, do your trading decisions change?

Do not just optimize your total PnL but also think about your risk-reward ratio. As the delta of an option is not constant, recalculate the total delta exposure of your position at each point in time and ensure it stays limited by hedging with stocks or other options. What is the cheapest hedge available?

Extra assignment: (Beta/Science students only)

1. Strike limits. For each option type and strike combination make sure have a maximum of 100 lots outstanding long/short.
2. Vega limits. At each point in time calculate the Vega of your whole position and make sure it stays below 4000 (long/short) by hedging with other options if needed.

Deliverables: A 3-6 page report describing your strategy. This needs to be relatively self-contained but does not need to re-explain concepts directly coming from the lectures. The report should contain at least the following, but may contain any other relevant information or visualisations:

A high-level description and/or visualisation of the dataset

Description of and argumentation for the strategies’ internal logic

A visualisation of the strategy performance, including:

The position taken per stock and option

The delta position of the stock, options and in total

The profit and loss over time per stock, option and in total

An appendix with the python code generating the strategies

(appendix does not count toward report pages)

Evaluation: Report (20%)

Analysis & Strategy (50%)

Analysis (20%)

Strategy: Potential, Description and Evaluation (20%)

Originality (10%)

Code Quality (30%)

Hints:

* Build up the code for a simple strategy first. Only after that code runs smoothly, tweak your strategy to optimize profit and risk-reward.
* When debugging the strategy, it helps to start with a smaller dataset by using only the first 250-1000 time periods, this will allow you to capture any bugs, visualize and tweak your strategy, while not having to wait a couple of minutes for each run as your algorithm gets more complex.

Guideline for basic Python implementation:

* Start by calculating the theoretical prices and greeks for each option and for each timestamp. Store them conveniently for lookup later, either alongside the market data or in a new DataFrame with the same index and similar column names. It will be easier to program the algorithm if you don't need to be concerned with calculating them inside your main strategy loop.
* Visualize the problem:
  + How are the prices of the stock and the prices of the options related?
  + How do the deltas of the options evolve? Can you explain the option price movements based on the delta?
  + How do your own prices relate to the market buying and selling prices? Is it clear when you would want to do a trade?
* Write the algorithm:
  + As before, because your algorithm will need to keep track of the previous positions, its behaviour will need to depend on the trades it did before. For this reason it will be easiest to implement the algorithm one instant at a time by looping over each of the observations and timestamps in the input data.
  + Inside the loop, for each step in time do the following:
    - Obtain the current available prices and volumes for the stock and the options, as well your previous position in each of them.
    - For each option:
      * Obtain your own pre-calculated theoretical price and delta.
      * Determine how much it costs to delta-hedge this option. If the delta is high (in absolute sense) you will need to buy/sell more stock against it. How expensive is this per lot of the option?
      * Use this information to determine what kind of trading offset you require on this option.
      * If the market ask is lower than your theoretical price by more than your trading offset requires, buy the option and hedge in the stock. Vice versa, if the market bid is higher than your theoretical price by more than the trading offset, sell the option and hedge in the stock. Is enough volume available in both the option and stock markets?
      * Keep track of how these trades influence your own position and available market volumes.
    - After applying all the trades, your full new position might still not be completely delta-hedged. The delta for each option does not remain constant over time, and hence the left-over position from earlier trades might have developed some deltas. Further, in doing your trades you might have had to round the delta exposures for each single option to trade a round amount of stocks. Are there any left-over delta positions for these reasons?
      * If so, hedge them if it is required to stay within your delta limit or if you want to reduce risk. (Check the available market volumes!)
* Finally, after you have determined your positions in the stock and each of the options for the whole dataset, use them to calculate your PnL At each instant, the PnL should consist of two parts:
  + The total money you have paid/received for the trades you have done so far
  + The valuation of the position you are holding